ENGTECH

WORKBOOK

# Why Do We Count in Tens? Place Value and Binary

#01 · Math from Zero

Exercises with fully worked solutions

17 exercises

**How to use this workbook**
Watch the video first. Work each exercise on the printed grid (or on squared paper) before you look at the solution, and check every result by undoing it. Getting one wrong is useful: find the exact step where it went off.
Numbers: from five digits up, we leave a thin space every three digits (1 250 000), the scientific style the videos use. In the US that gap is a comma, in Brazil a dot.

## Warm-up

**Recap:** Place value: reading from the right, each place is worth ten times the one before: ones, tens, hundreds, thousands. A zero holds an empty place open. Every three places form a period: ones, thousands, millions, billions, trillions. Each period after the first takes exactly three digits.

### Exercise 1

In the number 5082, how much is each digit worth? Why can't you just drop the 0, since it is worth nothing?

### Exercise 2

Swap the first and last digits of 614. What is the new number, and how much is the digit 6 worth in each?

### Exercise 3

A careless clerk drops the zero from 405 on a parts order. What number gets written, and how many parts go missing from the order?

### Exercise 4

Start from 2046. (a) Write a 0 on the right. (b) Instead, write a 1 on the left. (c) Instead, write a 0 on the left. What is the value each time?

### Exercise 5

Write in digits, marking the periods: forty million, nine thousand, twelve. Why does the thousands period need zeros, when nobody says them?

## Practice

**Recap:** Base: how many symbols each place has. Binary (base two) places are worth 1, 2, 4, 8, 16. Base six places: 1, 6, 36. Hexadecimal (base sixteen) places: 1, 16, 256, with A to F standing for ten to fifteen. Every four bits make one hex digit, because $2 \times 2 \times 2 \times 2 = 16$. The base is bigger than every digit. To compare different bases, convert to base ten first.

### Exercise 6

An odometer shows 1399. The car goes one more kilometer. What does it show now, and which wheels rolled over to zero?

### Exercise 7

A drive is rated at 2 trillion bytes, that is, $2 \times 10^{12}$ bytes: a 2 followed by 12 zeros. Write that number in digits. How many zeros does it have, and how many periods of three digits follow the 2?

### Exercise 8

Convert $10110_2$ to base ten.



### Exercise 9

Write 37 in binary, using the method of taking the biggest place that fits.



### Exercise 10

A chip holds $1011_2$. It writes a 0 on the right, then another 0. What number does it hold after each step, in base ten?



### Exercise 11

Turn the bits $10100111_2$ into two hex digits, using the four-bit shortcut. Then check the result in base ten.



### Exercise 12

Write $41_6$ (base six) in binary, passing through base ten.

### Exercise 13

What is the biggest number you can write with (a) 5 bits, (b) 3 hex digits? Answer in base ten, and show the rollover that comes right after.

### Exercise 14

(a) What is the smallest base in which the numeral 58 can exist? (b) Put these in order, smallest first: $1000_2$; $A_{16}$; $13_6$.

## Engineering challenge

**Recap:** A chip reads each bit from a voltage, with a safety gap between low and high. Noise margin: how far a signal can drift before it may be misread. A screen color is three pairs of hex digits, for red, green and blue, and each pair runs from 0 to $FF_{16}$ = 255.

### Exercise 15

Classic TTL logic chips use these limits. A driver guarantees a low output of at most 400 mV and a high output of at least 2400 mV. A receiver reads anything up to 800 mV as 0 and anything from 2000 mV up as 1. How much noise can a wire add on each state before the bit can be misread?

### Exercise 16

Picture a simplified flash cell with a voltage window of `3200` mV, split into equal bands, one band per level. Each extra bit doubles the levels. (a) How wide is one band for 3 bits and for 4 bits? (b) How many levels would 5 bits need, and how wide would a band be? (c) If each level sits in the middle of its band, how much noise can it tolerate in each case?



### Exercise 17

EngTech's positive green is the color code `2ECC71`: three pairs of hex digits, for red, green and blue. Find each amount in base ten. Which channel is strongest, and how far is it from the biggest value a pair can hold?

## Worked solutions

**Exercise 1**

Read the places from right to left: ones, tens, hundreds, thousands.

$5 \times 1000 = 5000$; $0 \times 100 = 0$; $8 \times 10 = 80$; $2 \times 1 = 2$.

Add them back to check: $5000 + 0 + 80 + 2 = 5082$.

The 0 holds the hundreds place open. Without it, the 5 slides down one place:

$5 \times 100 + 8 \times 10 + 2 = 582$.

> **Answer:** $5 \times 1000 = 5000$; $0 \times 100 = 0$; $8 \times 10 = 80$; $2 \times 1 = 2$.

**Exercise 2**

The original number: $6 \times 100 + 1 \times 10 + 4 = 614$, so the 6 is worth $6 \times 100 = 600$.

After the swap: $4 \times 100 + 1 \times 10 + 6 = 416$, so the 6 sits in the ones place and is worth $6 \times 1 = 6$.

Same digit, different place, different value.

> **Answer:** New number 416. The six goes from 600 to 6.

**Exercise 3**

The zero was holding the tens place open: $4 \times 100 + 0 \times 10 + 5 = 405$.

Without it, the 4 slides down into the tens: $4 \times 10 + 5 = 45$.

The order loses $405 - 45 = 360$ parts.

> **Answer:** It becomes 45; 360 parts go missing.

**Exercise 4**

(a) Every digit moves one place left, so the value is ten times bigger: $2046 \times 10 = 20\,460$.

(b) Nothing moves. The 1 fills the ten thousands place and adds its value: $10\,000 + 2046 = 12\,046$.

(c) Places are counted from the right, so a zero on the far left changes nothing: $0 \times 10\,000 + 2046 = 2046$.

> **Answer:** (a) 20 460; (b) 12 046; (c) 2046.

**Exercise 5**

Three periods: millions, thousands, ones. Every period after the first takes exactly three digits.

Thousands: nine becomes 009. Ones: twelve becomes 012.

$40 \times 1\,000\,000 + 9 \times 1000 + 12 = 40\,009\,012$.

Without those zeros the digits slide into the wrong places: $4 \times 10\,000 + 9 \times 100 + 12 = 40\,912$.

> **Answer:** 40 009 012

### Exercise 6

The ones wheel is on its last symbol, so it rolls to 0 and pushes the tens wheel.

The tens wheel is also on 9, so it rolls to 0 and pushes the hundreds wheel, which goes from 3 to 4.

$1399 + 1 = 1400$.

> **Answer:** 1400; the ones and tens wheels rolled over.

### Exercise 7

One trillion is a 1 followed by twelve zeros, so $2 \times 1\,000\,000\,000\,000 = 2\,000\,000\,000\,000$.

Twelve zeros split into periods of three: $4 \times 3 = 12$, so four periods follow the 2: billions, millions, thousands, ones.

> **Answer:** $2\,000\,000\,000\,000$: 12 zeros, in four periods of three.

### Exercise 8

From the right, the binary places are worth 1, 2, 4, 8, 16.

The 1s sit in the places worth 16, 4 and 2.

$10110_2 = 16 + 4 + 2 = 22$.

> **Answer:** 22

### Exercise 9

The places are 32, 16, 8, 4, 2, 1. The biggest that fits is 32: $37 - 32 = 5$.

16 and 8 don't fit, so two zeros. 4 fits: $5 - 4 = 1$.

2 doesn't fit, so a zero. 1 fits: $1 - 1 = 0$.

Check: $100101_2 = 32 + 4 + 1 = 37$.

> **Answer:** $100101_2$

### Exercise 10

Start: $1011_2 = 8 + 2 + 1 = 11$.

Each new 0 moves every bit up one place, so each bit is worth double: $10110_2 = 2 \times 11 = 22$.

Once more: $101100_2 = 2 \times 22 = 44$.

> **Answer:** 22, then 44.

### Exercise 11

Every group of four bits is one hex digit. Left group: $1010_2 = 8 + 2 = 10 = A_{16}$.

Right group: $0111_2 = 4 + 2 + 1 = 7 = 7_{16}$.

So the byte reads A7 in hex: $A7_{16} = 10 \times 16 + 7 = 167$.

Check straight from the bits: $10100111_2 = 128 + 32 + 4 + 2 + 1 = 167$.

> **Answer:** $A7_{16}$, which is 167.

### Exercise 12

The left place in base six is worth 6: $41_6 = 4 \times 6 + 1 = 25$.

Biggest place that fits is 16: $25 - 16 = 9$. Then 8 fits: $9 - 8 = 1$. 4 and 2 don't fit. 1 fits.

Check: $11001_2 = 16 + 8 + 1 = 25$.

> **Answer:** $11001_2$

### Exercise 13

(a) All wheels on the top symbol: $11111_2 = 16 + 8 + 4 + 2 + 1 = 31$. One more rolls over: $100000_2 = 31 + 1 = 32$.

(b) The hex places are worth 1, 16 and $16 \times 16 = 256$. All on F: $FFF_{16} = 15 \times 256 + 15 \times 16 + 15 = 4095$. One more rolls over: $1000_{16} = 4095 + 1 = 4096$.

Same pattern as 999 before 1000: the number right before a one followed by zeros is the biggest that fits.

> **Answer:** (a) 31, then $100000_2 = 32$; (b) 4095, then $1000_{16} = 4096$.

### Exercise 14

(a) The base must be bigger than every digit. The biggest digit is 8: $8 + 1 = 9$.

(b) Convert everything to base ten first: $1000_2 = 8$; $A_{16} = 10$; $13_6 = 1 \times 6 + 3 = 9$.

$1000_2 < 13_6 < A_{16}$.

> **Answer:** (a) Base 9. (b) $1000_2 < 13_6 < A_{16}$.

### Exercise 15

Low state, worst case: the driver sends 400 and the receiver accepts up to 800: $800 - 400 = 400$.

High state, worst case: the driver sends 2400 and the receiver accepts down to 2000: $2400 - 2000 = 400$.

This is called the noise margin. It holds only while the supply and temperature stay inside the chip's rated range.

> **Answer:** 400 mV on each state.

### Exercise 16

(a) 3 bits give 8 levels and 4 bits give 16. Split the window: $3200 \div 8 = 400$ and $3200 \div 16 = 200$.

(b) One more doubling: $16 \times 2 = 32$ levels. Split the window: $3200 \div 32 = 100$.

(c) From the middle of a band to its edge is half a band: $400 \div 2 = 200$; $200 \div 2 = 100$; $100 \div 2 = 50$. So the band width and the noise margin both halve with each bit. Real cells aren't this tidy, but the trade is real: more symbols, less margin.

> **Answer:** (a) 400 mV and 200 mV. (b) 32 levels, 100 mV each. (c) Half a band each time: 200; 100; 50 mV.

### Exercise 17

Red: $2E_{16} = 2 \times 16 + 14 = 46$.

Green: $CC_{16} = 12 \times 16 + 12 = 204$.

Blue: $71_{16} = 7 \times 16 + 1 = 113$.

The biggest pair is $FF_{16} = 15 \times 16 + 15 = 255$, so green is short by $255 - 204 = 51$.

> **Answer:** Red 46, green 204, blue 113. Green is strongest, 51 below the maximum.

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